

Confidence Limits of Evolutionary Synthesis Models^{*}

Miguel Cerviño

Max-Planck-Institut für Extraterrestrische Physik

Valentina Luridiana

Instituto de Astrofísica de Andalucía (CSIC)

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Abstract. The probabilistic nature of the IMF in stellar systems implies that clusters of the same mass and age do not present the same unique values of their observed parameters. Instead they follow a distribution. We address the study of such distributions in terms of their confidence limits that can be obtained by evolutionary synthesis models. These confidence limits can be understood as the inherent uncertainties of synthesis models. We will compare such confidence limits arising from the discreteness of the number of stars obtained with Monte Carlo simulations with the dispersion resulting from an analytical formalism. We give some examples of the effects on the kinetic energy, V-K, EW(H β) and multiwavelength continuum.

Keywords: Galaxies: evolution – Galaxies: statistics

1. Introduction

In recent years, several efforts have been dedicated to improve our understanding of stellar evolution with more detailed and complete theories; at the same time, more powerful observatories have been developed to test the theory. However, an intermediate tool is necessary to link these pieces of information when we deal with systems in which only the integrated light of stellar populations (and their by-products, like the emission line spectrum) is available: this tool are synthesis models. Recently, it has been established how the input libraries affect the predictions of synthesis models (see Bruzual, 2001 or Carigi, 2000 as examples). From the theoretical point of view, there are still several open questions in the modelization of stellar clusters by evolutionary synthesis codes. One of the most important ones is related with the conservation of energy and the Fuel Consumption Theorem established by Renzini & Buzzoni (1986) (see also Marigo & Girardi, 2001 for the link of chemical with spectrophotometric models). Other questions, related with “technical” details in the isochrones computation can be found in Cerviño et al. (2001). We also refer to the contribution of S. Yi in these proceedings.

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In addition to those listed, there is still a source of uncertainty arising from the use of the Initial Mass Function (IMF) and the effect of the discreteness in the number of stars in the models results (Buzzoni, 1989; Santos & Frogel, 1997; Lançon & Mouhcine, 2000; Cerviño et al., 2000; Plöschke, 2001; Bruzual, 2001; Cerviño et al., 2002). In this paper we present our current understanding of the dispersion introduced in the results of evolutionary synthesis models by the discreteness of the stellar population for a given IMF.

2. The modelization of real star forming regions

Using a careful analysis of the stars in the solar neighborhood, Kroupa (2001) shows that the Salpeter IMF is compatible with observations if stochastic effects are taken into account. Taking into account the discreteness of the stellar population, the predictions of any model that relies on an IMF are only exact *under the assumption of an infinite number of stars*. Otherwise, they only give a *mean value* of a probability distribution. The relevance of such fluctuations in the results of synthesis models is obvious in the case of massive stars and young clusters, *but they also affect the models of older clusters dominated by the emission of low-mass stars* since small variations in the initial mass/number of stars in a given mass range, can produce different numbers of, e.g., AGB stars at a given age, which in turn produce large variations in the resulting colors (see Santos & Frogel, 1997 and Bruzual, 2001 as examples).

So, for the comparison of models with observational data it is necessary to obtain not only the mean value of the observables, but also, at least, the corresponding dispersion of the computed observables due to the discreteness of the stellar population. Such dispersion can be evaluated theoretically in function of the effective number of stars, N_{eff} (Buzzoni, 1989; see also Cerviño et al., 2002):

$$\frac{\sigma_L}{\langle L \rangle} = \frac{1}{\sqrt{N_{\text{eff}}(L)}}, \quad (1)$$

As Buzzoni (1989) highlighted, N_{eff} is not a real number of stars, but rather a rough estimate of the number of stars contributing to a given variable. In Figures 1 to 3 we show several examples of the 90% confidence level of a large number of Monte Carlo simulation and their comparison with the relative dispersion obtained by the analytical formalism. All models assume an instantaneous burst of star formation and solar metallicity evolutionary tracks. Other examples can be found in Cerviño et al. (2000, 2001, 2002).

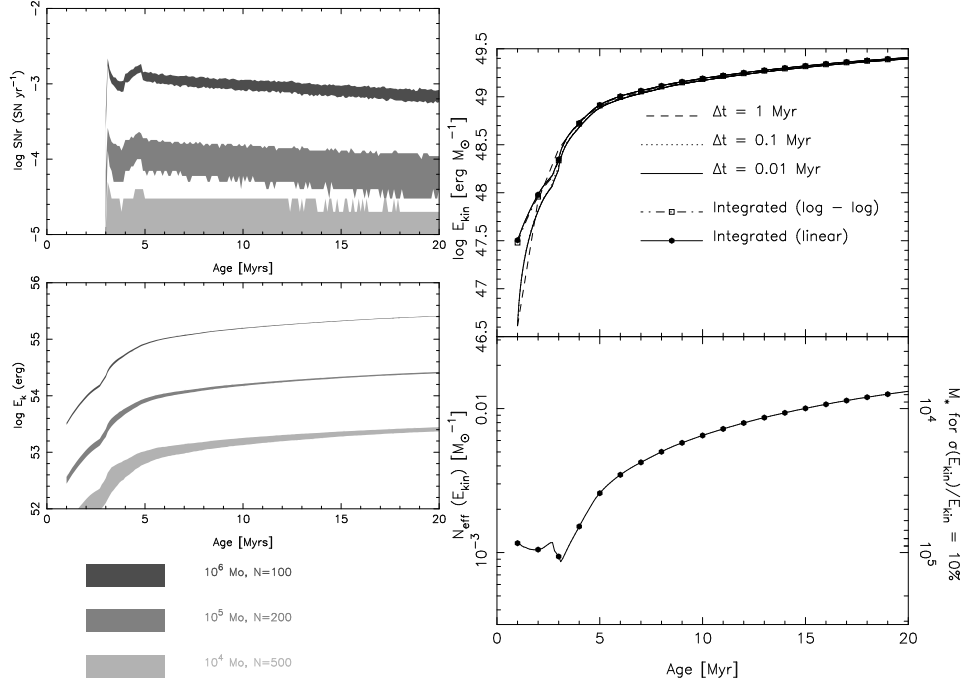


Figure 1. Left: Monte Carlo 90% Confidence Levels for the SN rate and the Kinetic for different amount of stars. Right: Analytical simulations and the corresponding $N_{\text{eff}}(E_K)$. Note that $N_{\text{eff}}(\text{SNrate}) = \text{SNr}$ by definition.

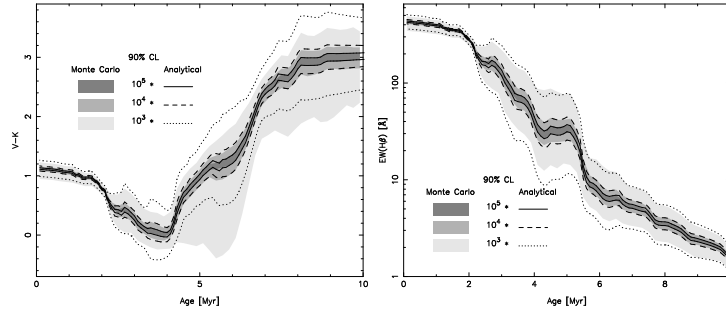


Figure 2. Monte Carlo and analytical 90% Confidence Levels for V-K and the EW(Hβ) in emission.

3. Conclusions

We have shown that the effects of fluctuations in the number of stars due to the stochastic nature of the stellar formation process and the discreteness of the stellar populations produce a dispersion in the predictions of evolutionary synthesis models, and that such dispersion may be much larger than the observational errors. The dispersion can be

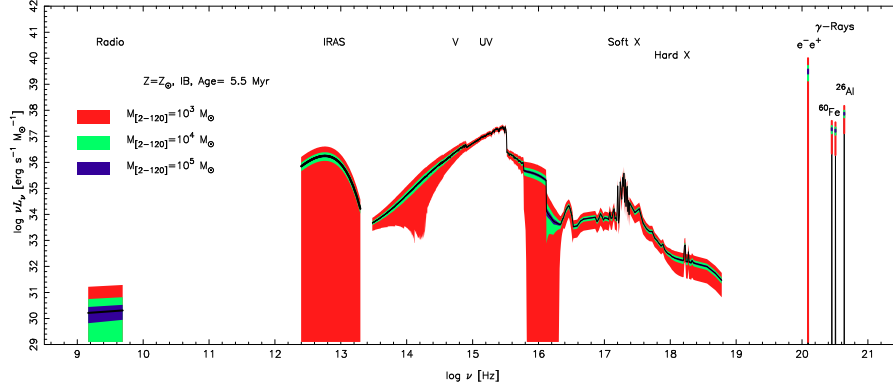


Figure 3. Analytical 90% Confidence Level for the multiwavelength spectrum for a 5.5 Myr old burst.

evaluated theoretically, and it can be used as an observable. The application of this ideas may help to improve the understanding of other astrophysical problems, for example: (i) Is it really necessary a IMF slope different from Salpeter's? (ii) Is it possible to explain the observed dispersion of chemical abundances by including the IMF fluctuations in chemical evolution models?. (iii) How much the underlying probability distribution of luminosities affects the corresponding colors?...

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